Math Maker Fair Math Explanation

Calculus, Web Development

* THIS IS THE DOCUMENT I WORKED FROM TO CREATE THE WEBSITE THIS PROJECT IS DISPLAYED ON.
* The website is [here.](https://pizza496.github.io/MMF-2020/)
* EVERYTHING AFTER THIS IS ON THE SITE IN SOME FORM (except unused content and source list).
* What is actually happening here?
  + What is being displayed is a set of vectors in the complex plane in the form where c is a complex number, and t is a variable for time. As time passes this vector will trace out a circle as the time goes from to
    - We’re using to represent the vectors because as t increases, the vector traces a circle, for the same reason .
      * For calculating for basic integers, we just multiply by itself repeatedly, but what if that value is pi?
      * Well looking back at the basic definition of e: where , if we raise that to pi, we get . So we’re going to take that fraction and multiply it by , and that gives us . Now to find the value, we have to increase the value of n, but then for each integer value of n, the total of is still a multiple of pi. The big but here is that eventually we’re going to increase n to infinity, so we don’t have to increase the power by factors of we can increase it by integers because those are easier, so we can say that where , and this is how the calculation for as well. Numerically, you increase m until the result is pretty accurate and it’s close enough for most purposes.
      * So why is true?
      * Well, using the previous definition for calculating powers of e arithmetically, .
        + The form of number is called a complex number, and multiplying them together works just like multiplying polynomials, . This can still be simplified further into because .
        + Multiplying these numbers on the complex plane has a fun little fact where if you make two triangle, abc and abd where a is at 0+0i and b is at 1+0i, and c is one of the two numbers you are multiplying and d is the other, if you take one triangle, and align side ab to side ac or ad, and scale the triangle so that the sides are equal in length, the c or d point on the scaled triangle will fall on the complex number equal to the product of the first two complex numbers. Below I’ve squared 1+1i to show you what I mean. The original triangle is in red and the rescaled triangle is in blue. This method says the product is 2i, algebraically the product is shown to be: .
        + If you raise these numbers to higher and higher powers, the triangles eventually form spiral like shapes unless the numbers fall on the unit circle (circle with radius 1, and centered on the origin in this case), where the triangles follow the circle around and around infinitely because side ab and side ac are both 1 unit long, and don’t need to be resized at all.
      * Back to , if we take this number and put it on the complex plane and do the multiplication out, for low values of m, the result isn’t very accurate, much like how calculating e with low values of n using the definition above isn’t very accurate. As we make m larger and larger approaching infinity, the initial complex number gets closer and closer to being on the unit circle. Once m reaches infinity the result turns out to be negative one. Before you go and type this into your calculator, it will probably fail. But if you take the approximation we found earlier and plug that in with increasing values for m, the result will approach -1+0i, or -1.
      * COMMENT
    - Now we need to understand why goes around the unit circle on the complex plane as t increases.
      * is just a specific form of , so what if we replaced t with ? Using the same approximation from earlier, this approaches 0+1i. Continuing to test different multiples of pi, we see that the distance from the origin to the result are all equal to 1, and if we put in some arbitrary value for t that isn't related to pi? Well it still holds true. If we put in 1 instead of t, the value approaches (when calculating cos and sin in radians). This means that for , t specifies the number of radians around the circle the vector has traveled.
    - So is a rotating vector that goes around in a circle as t goes from 0 to
    - To simplify things a little bit, I’m going to multiply the exponent by so that the vector traces a circle in 1 unit of t, not units.
    - Now we need the last piece of the vectors, the magnitude.
    - You can scale the magnitude of vectors by multiplying the vector by a constant, so this means if we want our vector to be 2 units long, we would want the vector . Generalizing it some more, we get that our vector is where c is equal to the magnitude of our vector.
  + Fourier Series
    - Some background: What we’re actually generating for these animations are Fourier Series that as time increases plot a function that can be any continuous line we want. A Fourier Series is defined as where is a complex number and is time. This function defines an infinite number of rotating vectors (see the vectors explanation) that as time progresses from to will make rotations. The challenge from here is limiting the series so that it can be calculated on a computer. Now the series for each animation is defined by a function like this: . Now we need to figure out what each coefficient is, and we do that by applying a Fourier Transform.
    - First: there are multiple types of Fourier Transforms, the basic Fourier Transform is defined by the equation for any real number . This will take a function and decompose it into a set of periodic functions, similar to how a musical chord can be broken down into notes. In these animations the periodic functions we break the function down into are the rotating vectors. For my purposes, a continuous Fourier Transform isn’t that applicable because my sample size for the “function” isn’t infinite, so I will be using a Discrete Fourier Transform.
      * This transforms a sequence of N complex numbers into another sequence of complex numbers
      * Example

because the number of samples in this example is 4

* + - For the animations, I’m calculating the discrete Fourier transform of the function, but over sample sizes in the thousands instead of four.
* Process
  + Take an SVG image
  + Express the curves as a function with real input (time) and a complex output
    - * Where c\_n is some complex number, t is a value of time
  + Take sample outputs of the function over the range of time
  + Take the Fourier transform of the samples
* Things that need to be understood
  + Fourier transform
  + Fourier series
  + Vectors and ways to write them
    - * Complex plane
    - etc.
* Sources
  + 3Blue1Brown
    - [Fourier Series Video](https://www.youtube.com/watch?v=r6sGWTCMz2k)
    - [e^(ipi) Video 3B1B](https://www.youtube.com/watch?v=v0YEaeIClKY)
    - [Fourier Transform Video](https://www.youtube.com/watch?v=spUNpyF58BY)
  + The Coding Train
    - [Fourier Series Pt. 1 Video](https://www.youtube.com/watch?v=Mm2eYfj0SgA)
    - [Fourier Series Pt. 2 Video](https://www.youtube.com/watch?v=MY4luNgGfms)
  + Mathlogger
    - [Fourier Series Video ML](https://www.youtube.com/watch?v=qS4H6PEcCCA)
    - [e^(ipi) Video ML](https://www.youtube.com/watch?v=-dhHrg-KbJ0)
  + Anderstood
    - [Another Person's Method (Mathematica Stack Exchange)](https://mathematica.stackexchange.com/questions/171755/how-can-i-draw-a-homer-with-epicycloids)
  + Phil Danne
    - [Source of Base Code For Animation](https://www.tomesoftware.com/labs/using-fourier-series-draw-svg-images/)
* Unused Content
  + - Why and not some other way to represent vectors?
      * Short answer: the math ends up being cleaner, because of the same reason
      * Taking the derivative (fancy word for finding the slope of a function at any given point) of results in , this means that the slope of a graph of at any t value, is equal to at that t value. If you give t a coefficient the derivative is multiplied by that coefficient .
      * What if the coefficient is ?
        + Then the derivative is .
      * First let’s imagine a particle whose position is defined by the equation , and that this particle is placed on the complex plane where the horizontal axis is represented by real numbers and the vertical axis is represented by imaginary numbers. If , , meaning the position of this particle is 1, and the derivative (velocity): , when graphed on the complex plane using vectors to represent the position and velocity values, (see below), the velocity of the particle is at a right angle to the particle’s position vector.
      * Now remember that the derivative for is always . This means that the velocity of the particle will always be at a right angle to the position vector. This means that the only way for t to increase is if the position vector traces a circle because that is the only time when